

Sample Test Medical Systems ST-1

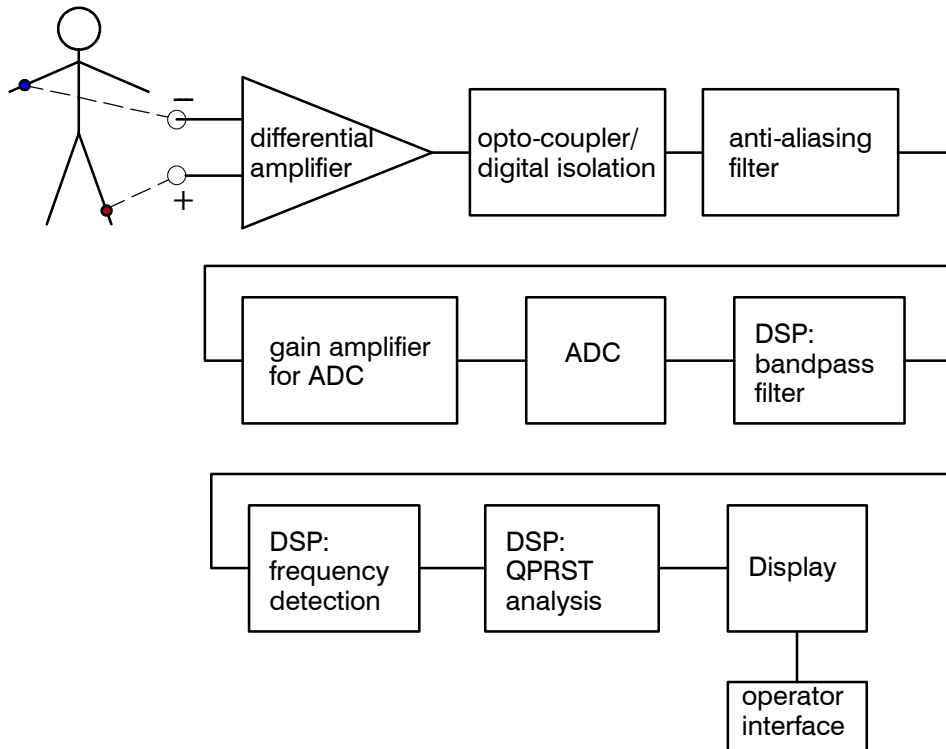
Time 120 minutes
– class documents allowed –

Solutions

(1) Biosignal Chain

Components of a portable ECG device (one channel)

(1.1)



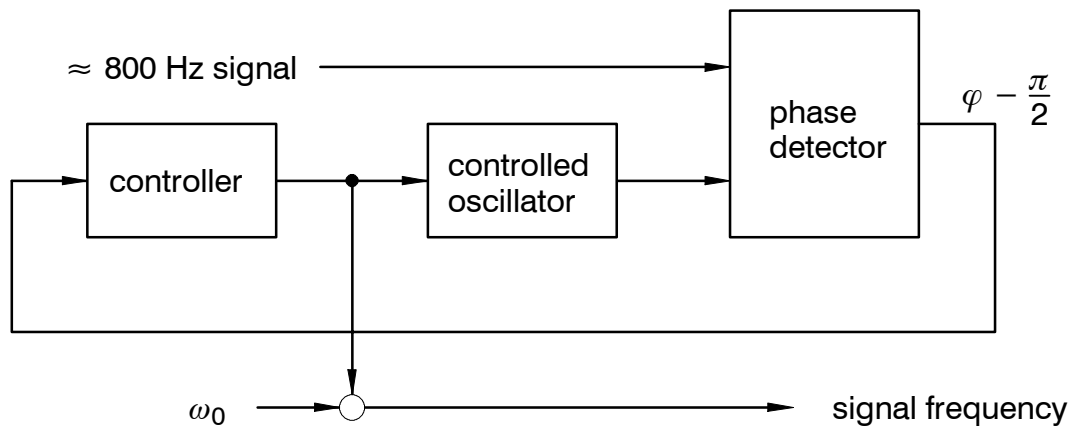
(1.2) Analog: anti-aliasing filter ($< 0.5 * \text{sampling frequency}$)
digital: bandpass filter ($\approx 0.04\text{Hz} \dots 150 \text{Hz}$).

(1.3) Possible resolution is 16 bits or better. 16 bits offer 15 bits in magnitude (1 sign bit). Every bit increases dynamic range by 6 dB, the dynamic range is 90 dB.

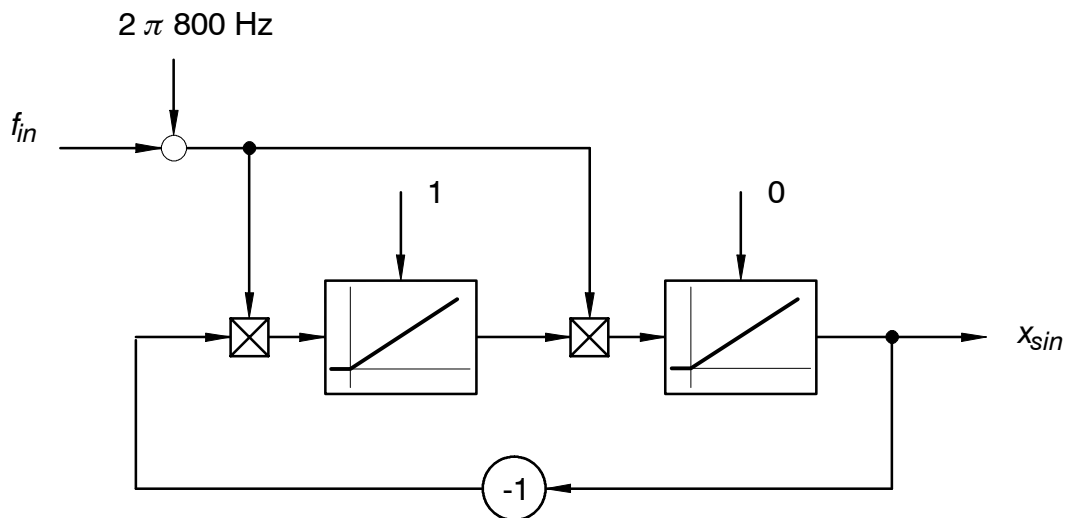
(1) = 6	(2) = 6	(3) = 6	(4) = 6	(5) = 6	$\Sigma = 30$				
1.0 \geq 29	1.3 \geq 27	1.7 \geq 26	2.0 \geq 24	2.3 \geq 23	2.7 \geq 21	3.0 \geq 20	3.3 \geq 18	3.7 \geq 17	4.0 \geq 15

(2) PLL for 800 Hz Frequency Tracking

(2.1)



(2.2)



One of the integrators has to be initialized by '1', the other one with '0'.

(2.3) The low pass filter has to reject the 1.6 kHz product in the phase detector. In order to avoid excessive phase lag for the controller a 2. order lowpass can be used. A reasonable cutoff frequency can be 400 Hz or 200 Hz (it does not affect the functionality). A 300 Hz lowpass with real poles (aperiodic) has transfer

$$\text{function } G_{filt} = \frac{1}{(T_1 s + 1)^2} = \frac{1}{T_1^2 s^2 + 2T_1 s + 1}.$$

The time constant is $T_1 = \frac{1}{\omega_c} = 5.3052 \cdot 10^{-4}$.

- (2.4) The transfer function of the process is $G = \frac{1}{2s}$ (the factor 2 comes from the phase detector). Basically it integrates a frequency difference to a phase difference (integrator). This is only true if the control loop is slow with respect to the cutoff frequency of the lowpass filter (here 300 Hz).

The PI controller has transfer function $K = k_p + \frac{k_i}{s}$. Comparing the closed loop transfer function with the normal form (2. order) gives $k_i = 2\omega_0^2$ and $k_p = \sqrt{8k_i} = 4\omega_0$. With $\omega_0 = 2\pi \cdot 10$ we obtain $k_p = 251.3$ and $k_i = 7896$.

(3) Butterworth Highpass IIR filter

- (3.1) The 3. order Butterworth lowpass is given by $G_b = \frac{1}{1 + 2s_n + 2s_n^2 + s_n^3}$.

Here $s_n = \frac{s}{2\pi \cdot 10}$. Thus the coefficients are $G_b = \frac{1}{\frac{s^3}{w_c^3} + \frac{2s}{w_c} + \frac{2s^2}{w_c^2} + 1}$.

- (3.2) The lowpass to highpass conversion is done by replacing $s_n \rightarrow \frac{1}{s_n}$. We get by replacing s_n with s

$$G_{b_high} = \frac{\frac{s^3}{w_c^3}}{\frac{s^3}{w_c^3} + \frac{2s}{w_c} + \frac{2s^2}{w_c^2} + 1} = \frac{4.03 \cdot 10^{-6} s^3}{4.03 \cdot 10^{-6} s^3 + 5.07 \cdot 10^{-4} s^2 + 0.032 s + 1}$$

- (3.3) The filter can be converted by Z-transform or (better) by bilinear transform. Sampling frequency should be at least 10 times the corner frequency ($f_{sample} > 100$ Hz).

(4) 8-Tap Bandpass FIR Filter

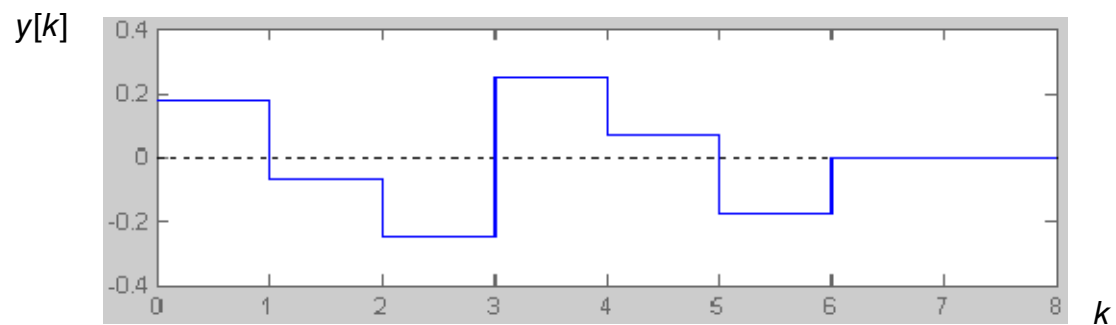
- (4.1) With 1 kHz sampling frequencies the discrete frequencies are $m \cdot f_s / 8$ 0Hz–125Hz–250Hz–375Hz–500Hz. Thus $H(m)$ becomes $H = [1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0]$ (250Hz and 375Hz are made equal to '1', since the 300Hz discrete frequency does not exist). The inverse FFT becomes real because $H(m)$ is symmetric.

- (4.2) Since the last element is of $h(k)$ is zero, the order of $G_{FIR}(z)$ becomes reduced.

$$G_{FIR(z)} = \frac{0.1768z^6 - 0.25z^5 - 0.1768z^4 + 0.5z^3 - 0.1768z^2 - 0.25z + 0.1768}{z^6}$$

(4.3) The step response is the sum of the values of the impulse response:

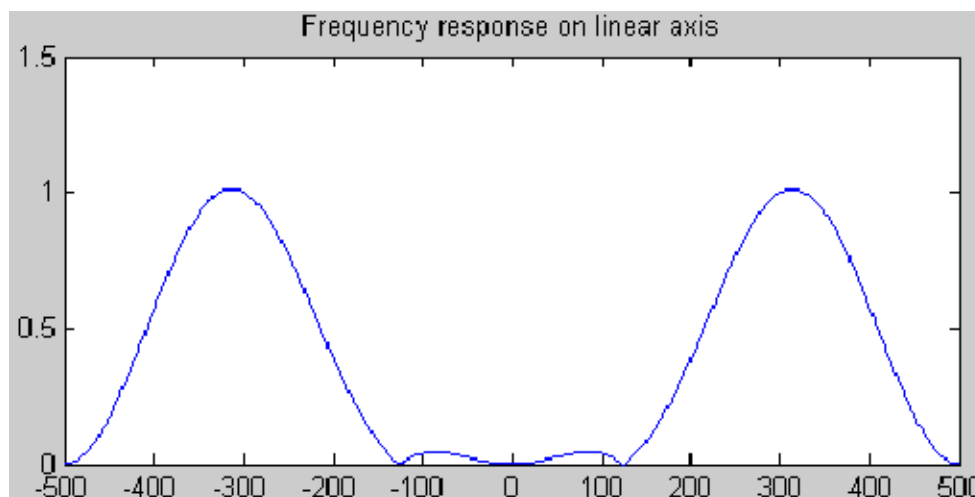
$$y[k] = [0.1768 \quad -0.0732 \quad -0.2500 \quad 0.2500 \quad 0.0732 \quad -0.1768 \quad 0 \quad 0 \dots]$$



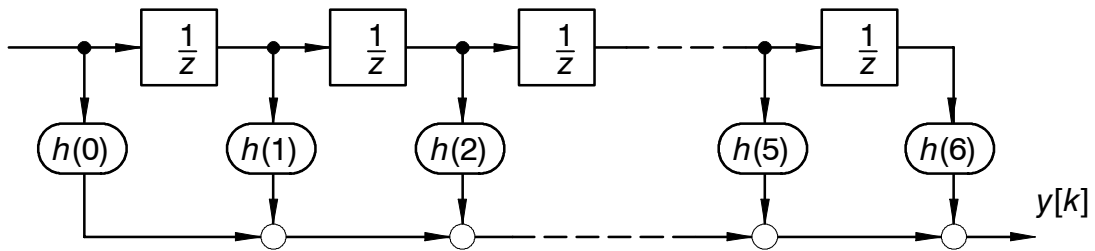
The dc-gain $\lim_{z \rightarrow 1} G_{FIR}(z) \stackrel{!}{=} 0$ must be satisfied for a bandpass, the sum of the numerator coefficients is exactly zero (can be also seen from the step response for $k \geq 6$).

(4.4) Since 300Hz is not a discrete frequency for an 8 tap FIR filter with 1kHz sampling frequency the maximum will occur between 250Hz and 375Hz. Since the gain of is exactly equal to '1' for 250Hz and for 375Hz the 300Hz gain is greater (approx. 1.3 according to the frequency response plot).

if all coefficients of the numerator are reduced by the factor 1/1.3 a gain of '1' is achieved for 300 Hz (see below). It is also possible to adjust $H(m)$.



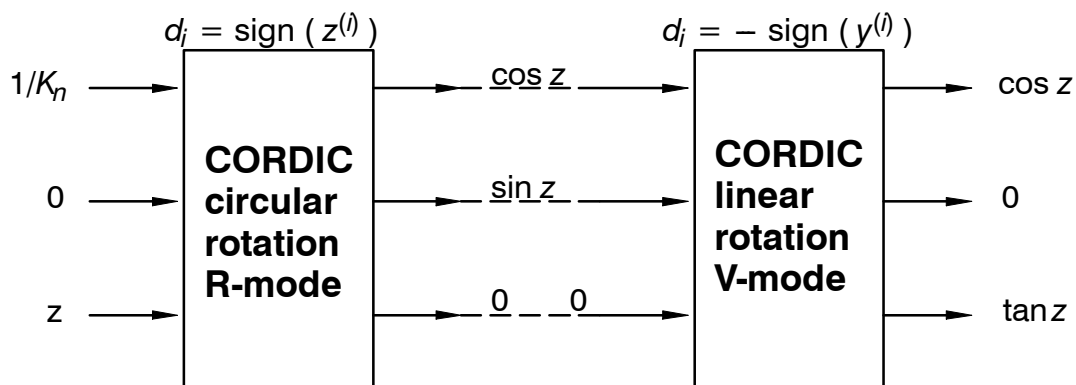
(4.5) Other forms of the FIR block diagram are also possible.



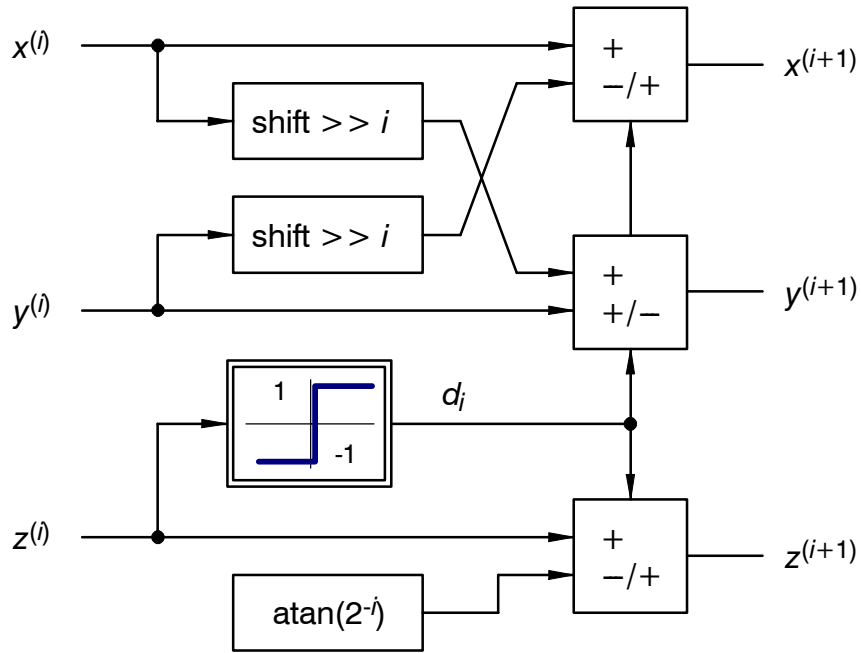
(4.6) Increasing the FIR order give better performance and sharper frequency response. Windowing techniques (Blackman, Kaiser-Bessel) reduce stopband and passband ripples.

(5) CORDIC Tangent Computation

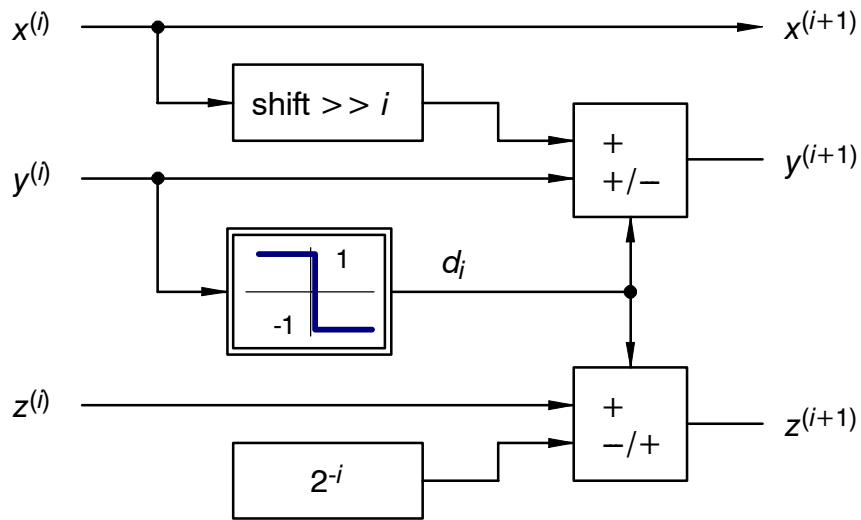
(5.1) Block diagram contains two CORDIC blocks (sine / cosine and division). The CORDIC algorithm requires only shift, add and subtract blocks.



(5.2) CORDIC rotation cell and CORDIC vectoring cell:



Rotation cell (circular rotation)



Vectoring cell (linear rotation)

(5.3) The maximum angle error is $\delta = \text{atan}(2^{-(10-1)}) = 0.112^\circ$

The maximum value occurs when $\sin(x)$ is maximal and $\cos(x)$ is minimal, thus

$$\frac{\sin(45 + \delta)}{\cos(45 + \delta)} = 1.0039 . \text{ Similar the minimum value is } \frac{\sin(45 - \delta)}{\cos(45 - \delta)} = 0.9961 .$$

The division computation has maximum error of $2^{-9} = 0.002$ which is added or subtracted in the last stage.

$$\Rightarrow \tan(45^\circ)_{\min} = 1.0039 + 0.002 = 1.0059 \quad (+0.59\% \text{ error})$$

$$\Rightarrow \tan(45^\circ)_{\max} = 0.9961 - 0.002 = 0.9941 \quad (-0.59\% \text{ error})$$