

## (1) Boolean Logic

(1.1) DeMorgan Theorem

The boolean equation $y=x_{1} \cdot x_{2}{ }^{\prime} \cdot x_{3}{ }^{\prime} \cdot x_{4}$ should be converted to OR/NOR logic by applying the DeMorgan theorem.
Draw the complete circuit diagram with OR/NOR gates.
(1.2) Duality Theorem

Apply the Duality theorem to $(x+0) \cdot(x+1)=x$ (assume it is correct).
Prove the equation after applying the Duality theorem.
(1.3) Logic Minimization

Find the minimum product of sums solution for the following truth table

| a | b | c | y |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |.

Draw the complete circuit diagram using only NOR gates!

(1.4) Hazards

The combinational circuit $y=x_{2} \cdot x_{1}{ }^{\prime}+x_{3}{ }^{\prime} \cdot x_{1} \cdot x_{0}$ might produce erroneous output (hazard). Draw the circuit with inverters, AND and OR gates. Draw a timing diagram for the change from $x_{3}={ }^{\prime} 0 ', x_{2}=' 1$ ', $x_{1}={ }^{\prime} 1 ', x_{0}={ }^{\prime} 1$ ' to $x_{3}=$ '0', $x_{2}=$ ' 1 ', $x_{1}=$ ' 0 ', $x_{0}=$ ' 1 '. What type of hazard occurs?
Change the design so that it becomes Hazard-free.

## (2) Hardware State Machine (Moore Machine)

The following signals should be generated by a state machine with every clock cycle. Inputs of the state machine:
reset $\quad{ }^{\prime} 1$ ' $=>$ reset of the state machine, ' 0 ' operation of sm,
start $\quad{ }^{\prime} 1$ ' $=>$ generate sequence below, ' 0 ' $=>y_{0}=y_{1}={ }^{\prime} 0$ '.

(2.1) How many states are required; how many flip-flops are required?
(2.2) Draw the state diagram.
(2.3) Design minimal solutions for the next-state logic.
(2.4) Design minimal solution for the output logic.
(2.5) Draw the flip-flops and output logic hardware circuit.

## (3) Coupled Flip-Flops

The following circuit should be synthesized by VHDL.

(3.1) Synthesize the circuit as structural design in VHDL. All gates and flip-flops should be seen in the VHDL code.
(3.2) What could be the purpose of this digital system?
(3.3) Design the same function in a behavioral way (using a standard_logic_vector for all flip-flops).

## (4) AC Machine Power Transistor Signals

Controlled AC drives (induction or synchronous machines) require a power inverter.


The principle of operation is shown in the above figure where pairs of power transistors are represented by switches that can be in a ' 0 ' and a ' 1 ' position. The switch position can be seen as a boolean signal.

The inverter is in "off"-state for $S_{3}=S_{2}=S_{1}={ }^{\prime} 0$ '.
Positive rotation of the machine occurs for the sequence:

| S3 | S2 | S1 |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 1 | 0 |
|  | $\cdots$ |  |
|  | $\cdots$ |  |
|  |  |  |

reset
start_stop
rl_time
${ }^{\prime} 1^{\prime}=>$ resets the state machine
'1' => normal (rotation) operation, '0' => "off"-state.
$=>$ determines machine frequency. This input
signal is 10 bits and it determines the number of clk cycles from one switch position to the next.
(4.1) Design a state diagram (should be consistent with your VHDL code) containing states, inputs and outputs.
(4.2) Write a VHDL module that creates the sequence of output signals as a timed state machine. Use the reset and the start_stop and the rl_time signals.
(4.3) What needs to be changed to reverse the sequence of the switches $S_{3}, S_{2}$, and $S_{1}$ ?

