

Sample Test Digital Systems / VHDL ST-1

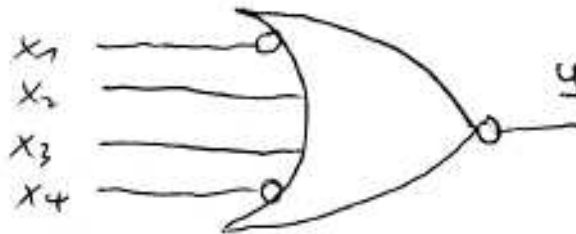
Time 120 minutes
– class documents allowed –

Solutions

(1) Boolean Logic

(1.1) DeMorgan Theorem

$$y = x_1 \cdot x_2' \cdot x_3' \cdot x_4 = y'' = (x_1' + x_2 + x_3 + x_4)'$$



(1.2) Duality Theorem

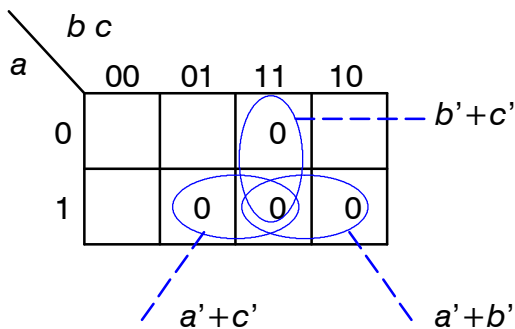
Apply the Duality theorem to $(x + 0) \cdot (x + 1) = x$

$$x \cdot 1 + x \cdot 0 = x \quad (\text{no parentheses required})$$

$$x \cdot 1 + x \cdot 0 = x + 0 = x, \quad \text{q.e.d.}$$

(1.3) Logic Minimization

product of sums means “zeros”

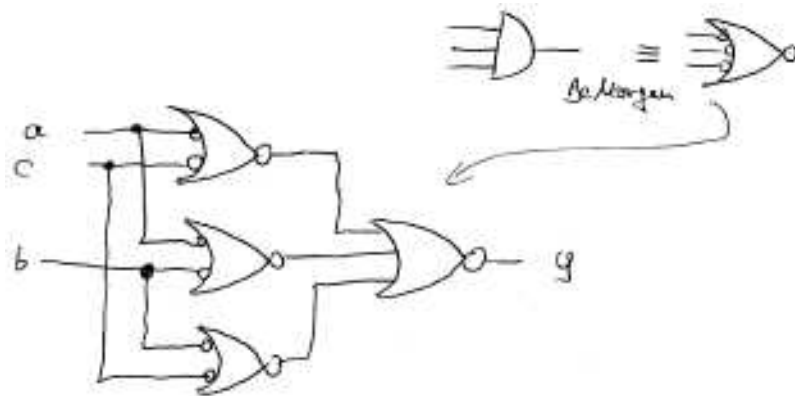


$$y = (a'+c') \cdot (a'+b') \cdot (b'+c')$$

Complete circuit diagram with NOR gates (DeMorgan was applied here):

| | | | | | | | | | |
|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| (1) = 6 | (2) = 6 | (3) = 6 | (4) = 6 | (5) = 6 | $\Sigma = 30$ | | | | |
| 1.0 \geq 29 | 1.3 \geq 27 | 1.7 \geq 26 | 2.0 \geq 24 | 2.3 \geq 23 | 2.7 \geq 21 | 3.0 \geq 20 | 3.3 \geq 18 | 3.7 \geq 17 | 4.0 \geq 15 |

Digital Systems / VHDL

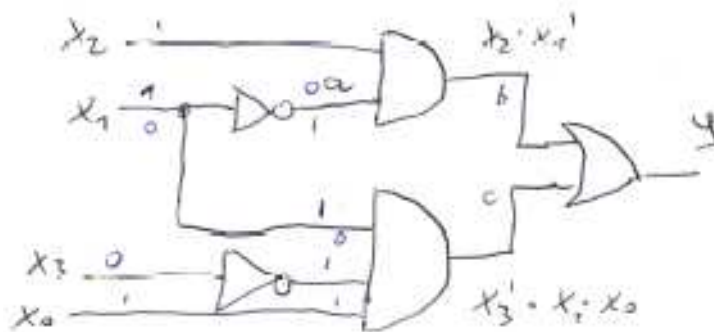


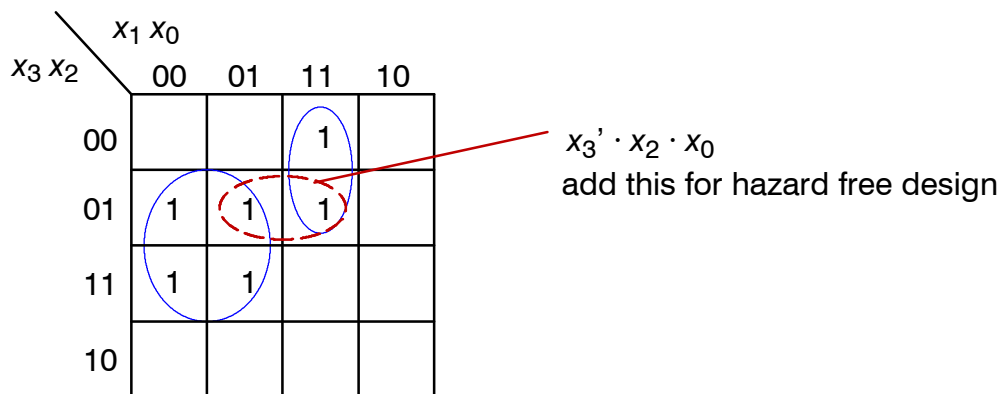
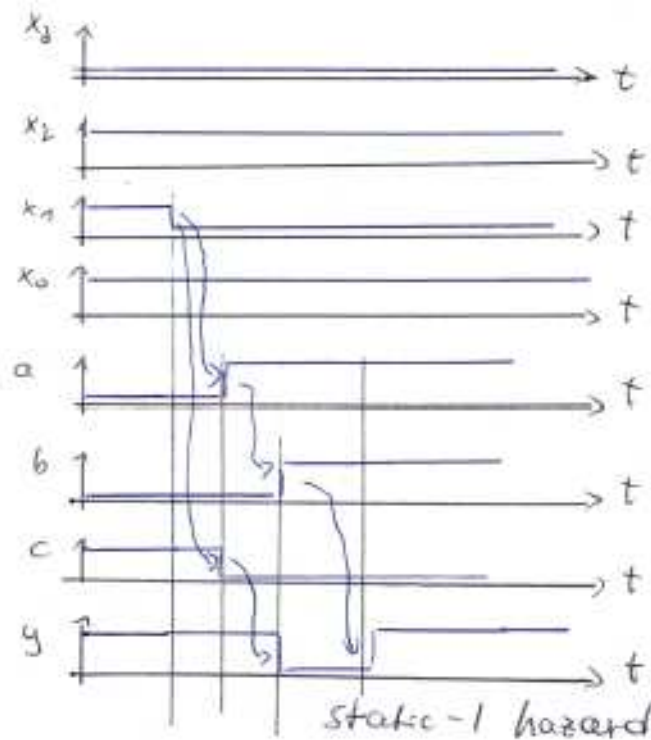
(1.4) Hazards

| x3 | x2 | x1 | x0 | y |
|----|----|----|----|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

<--+
|
--+

(in both cases output is '1')





Hazard-free version:

$$y = x_2 \cdot x_1' + x_3' \cdot x_1 \cdot x_0 + x_3' \cdot x_2 \cdot x_0 \text{ (one additional AND).}$$

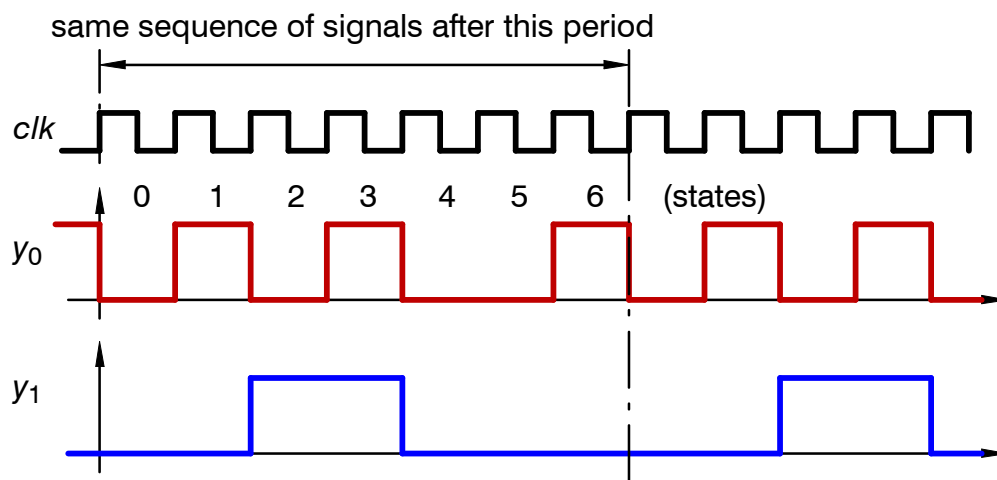
(2) Hardware State Machine (Moore Machine)

The inputs *reset* and *start* have a similar effect, so a new signal

$$operate = reset \cdot start$$

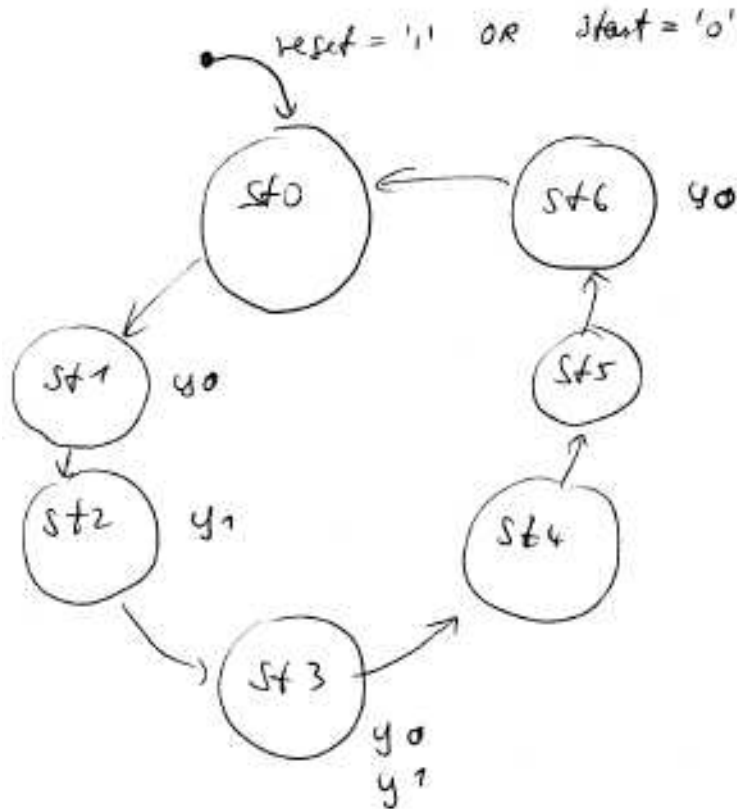
can be used to create a (simpler) next state logic.

| operate | x2 | x1 | x0 | x2n | x1n | x0n | (next states) |
|---------|----|----|----|-----|-----|-----|---------------|
| 0 | x | x | x | 0 | 0 | 0 | |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 | |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 | |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | |
| 1 | 1 | 1 | 1 | x | x | x | (don't cares) |



(2.1) State machine has 7 different states; 3 FFs are required (would allow 8 states).

(2.2) State diagram



(2.3) Next-state logic (from above truth table)

| Op x ₂ | x ₁ x ₀ | | | |
|-------------------|-------------------------------|----|----|----|
| | 00 | 01 | 11 | 10 |
| 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 1 | 1 | X | 0 |
| 10 | 0 | 0 | 1 | 0 |

x_{2n}

$$x_{2n} = Op \cdot (x_1' + x_0) \cdot (x_2 + x_1)$$

| | | | | | |
|----------|----|-----------|----|----|----|
| | | $x_1 x_0$ | | | |
| $Op x_2$ | | 00 | 01 | 11 | 10 |
| | 00 | 0 | 0 | 0 | 0 |
| | 01 | 0 | 0 | 0 | 0 |
| | 11 | 0 | 1 | X | 0 |
| | 10 | 0 | 1 | 0 | 1 |

x1n

$$x1n = Op \cdot x1' \cdot x0 \\ + Op \cdot x2' \cdot x1 \cdot x0'$$

| | | | | | |
|----------|----|-----------|----|----|----|
| | | $x_1 x_0$ | | | |
| $Op x_2$ | | 00 | 01 | 11 | 10 |
| | 00 | 0 | 0 | 0 | 0 |
| | 01 | 0 | 0 | 0 | 0 |
| | 11 | 1 | 0 | X | 0 |
| | 10 | 1 | 0 | 0 | 1 |

x0n

$$x0n = Op \cdot x0' \cdot (x2' + x1' + x0)$$

(2.4) Output logic

| x_2 | x_1 | x_0 | y_0 | y_1 |
|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | x | x |

| | | | | | |
|-------|---|-----------|----|----|----|
| | | $x_1 x_0$ | | | |
| x_2 | | 00 | 01 | 11 | 10 |
| | 0 | 0 | 1 | 1 | 0 |
| | 1 | 0 | 0 | X | 1 |

y0

$$y0 = x2' \cdot x0 + x2 \cdot x1$$

(3.1) *Structural design* in VHDL

```
entity prbs_struct is
  Port ( u : in  STD_LOGIC;
        clk : in  STD_LOGIC;
        x : out STD_LOGIC);
end prbs_struct;

architecture Behavioral of prbs_struct is
  SIGNAL a, b, q0, q1, q2, q3: std_logic;
begin

  x <= q3;
  a <= q2 XOR q3;
  b <= u OR a;

  ffs: PROCESS(clk, u)
  BEGIN
    IF clk'event AND clk='1' THEN
      q0 <= b;
      q1 <= q0;
      q2 <= q1;
      q3 <= q2;
    END IF;
  END PROCESS ffs;

end Behavioral;
```

(3.2) Purpose: Signal generation (Pseudo random binary sequence = PRBS)

(3.3) Behavioral design

```
entity prbs_behav is
  Port ( u : in  STD_LOGIC;
         clk : in  STD_LOGIC;
         x : out STD_LOGIC);
end prbs_struct;

architecture Behavioral of prbs_behav is
  SIGNAL qff : std_logic_vector(3 downto 0);
begin

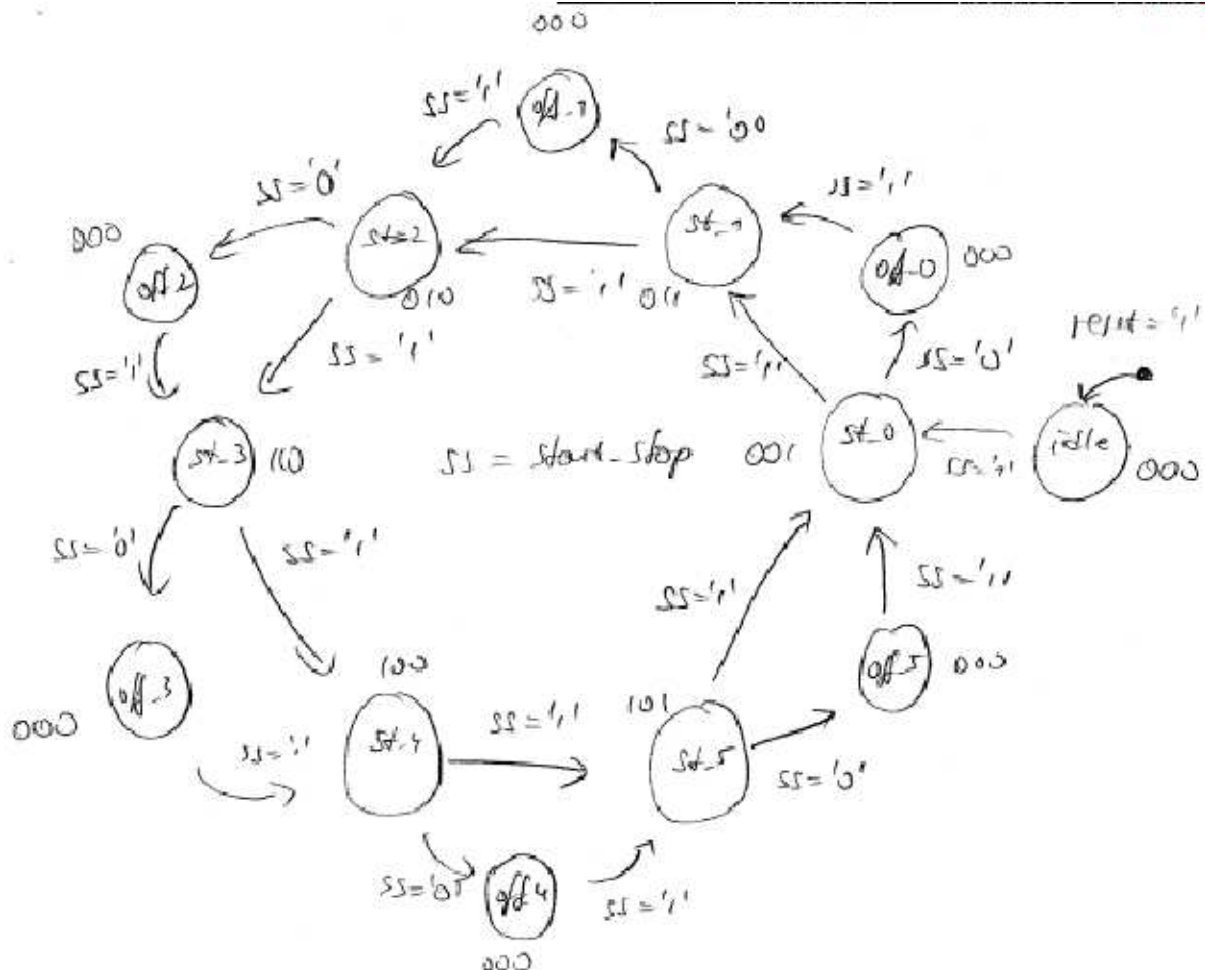
  x <= qff(3);

  ffs: PROCESS(clk, u)
  BEGIN
    IF clk'event AND clk='1' THEN
      if u='1' THEN
        qff <= qff(2 downto 0) & '1';
      ELSE
        qff <=
          qff(2 downto 0) & (qff(2) XOR qff(3));
      END IF;
    END IF;
  END PROCESS ffs;

end Behavioral;
```

(4) AC Machine Power Transistor Signals

(4.1) State diagram:



(4.2) VHDL module:

```

library IEEE;
use IEEE.STD_LOGIC_1164.ALL;
use IEEE.STD_LOGIC_UNSIGNED.ALL;

entity inverter is
  Port ( reset : in  STD_LOGIC;
        start_stop : in  STD_LOGIC;
        rl_time : in  STD_LOGIC_VECTOR(9 DOWNTO 0);
        clk : in  STD_LOGIC;
        STrans : out  STD_LOGIC_VECTOR (2 downto 0));
end inverter;

```

```

architecture Behavioral of inverter is
TYPE state_type IS (idle, st_0, st_1, st_2, st_3, st_4, st_5,
                    off_0, off_1, off_2, off_3,
off_4, off_5);
SIGNAL state, next_state : state_type;
begin

```

```

-- timed state machine
seq : PROCESS(clk, reset, next_state, rl_time)
VARIABLE cnt : INTEGER RANGE 0 to 1023 := 0;
BEGIN
    IF clk'event AND clk='1' THEN
        IF reset='1' THEN
            state <= idle;
            cnt := 0;
        ELSE
            cnt := cnt + 1;
            IF cnt=rl_time THEN
                state <= next_state;
                cnt := 0;
            END IF;
        END IF;
    END IF;
END PROCESS seq;

```

```

cmb: PROCESS(state, start_stop)
BEGIN
    next_state <= state;
    STrans <= "000"; -- default
    CASE state IS
        WHEN idle =>
            IF start_stop='1' THEN
                next_state <= st_0;
            END IF;
        WHEN st_0 =>
            IF start_stop='1' THEN
                next_state <= st_1;
            ELSE
                next_state <= off_0;
            END IF;
            STrans <= "001";
        WHEN st_1 =>
            IF start_stop='1' THEN

```

```
        next_state <= st_2;
    ELSE
        next_state <= off_1;
    END IF;
    STrans <= "011";
WHEN st_2 =>
    IF start_stop='1' THEN
        next_state <= st_3;
    ELSE
        next_state <= off_2;
    END IF;
    STrans <= "010";
WHEN st_3 =>
    IF start_stop='1' THEN
        next_state <= st_4;
    ELSE
        next_state <= off_3;
    END IF;
    STrans <= "110";
WHEN st_4 =>
    IF start_stop='1' THEN
        next_state <= st_5;
    ELSE
        next_state <= off_4;
    END IF;
    STrans <= "100";
WHEN st_5 =>
    IF start_stop='1' THEN
        next_state <= st_0;
    ELSE
        next_state <= off_5;
    END IF;
    STrans <= "101";
WHEN off_0 =>
    IF start_stop='1' THEN
        next_state <= st_1;
    END IF;
WHEN off_1 =>
    IF start_stop='1' THEN
        next_state <= st_2;
    END IF;
WHEN off_2 =>
    IF start_stop='1' THEN
```

```
                next_state <= st_3;
            END IF;
        WHEN off_3 =>
            IF start_stop='1' THEN
                next_state <= st_4;
            END IF;
        WHEN off_4 =>
            IF start_stop='1' THEN
                next_state <= st_5;
            END IF;
        WHEN off_5 =>
            IF start_stop='1' THEN
                next_state <= st_0;
            END IF;
    END CASE;
END PROCESS cmb;

end Behavioral;
```

- (4.3) Reversing the sequence of the switches requires an addition input, i.e. "direction". The IF clauses in the combinational PROCESS need to be modified, for instance:

```
        WHEN st_2 =>
            IF start_stop='1' THEN
                IF direction='0' THEN
                    next_state <= st_3;
                ELSE
                    next_state <= st_1;
                ELSIF direction='0' THEN
                    next_state <= off_2;
                ELSE
                    next_state <= off_1;
                END IF;
            STrans <= "010";
```