

Sample Test PS-DIGS W14/15

Time 90 Minutes
– Use of class documents allowed –

Solutions

(1) Conversion of Number Systems and Dual Number Computation

(1.1) Maximum positive number $i_{max} = 2^{16} - 1 = 32767$, minimum (negative) number is $i_{min} = -2^{16} = -32768$ (16 bits = 15 bits for representing magnitude, 1 sign bit).

(1.2)

```

348 : 2 = 174,    rem = 0
174 : 2 = 87,     rem = 0
87  : 2 = 43,     rem = 1
43  : 2 = 21,     rem = 1
21  : 2 = 10,     rem = 1
10  : 2 = 5,      rem = 0
5   : 2 = 2,      rem = 1
2   : 2 = 1,      rem = 0
1   : 2 = 0,      rem = 1    (end of algorithm)
⇒ x = 1010111002 (from remainders)
    
```

(1.3) Representation of $-x$

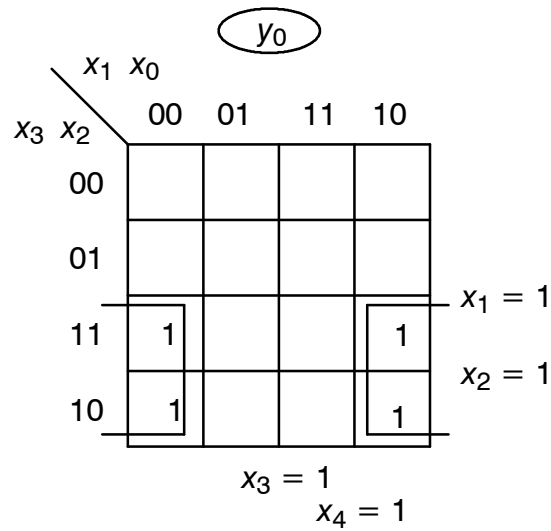
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0000 0001 0101 1100
1111 1110 1010 0011  (inversion)
+                               1  (add of 1)
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1111 1110 1010 0100  (2's complement of x)
    
```

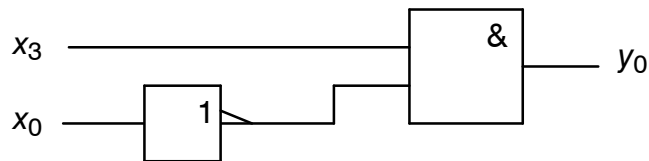
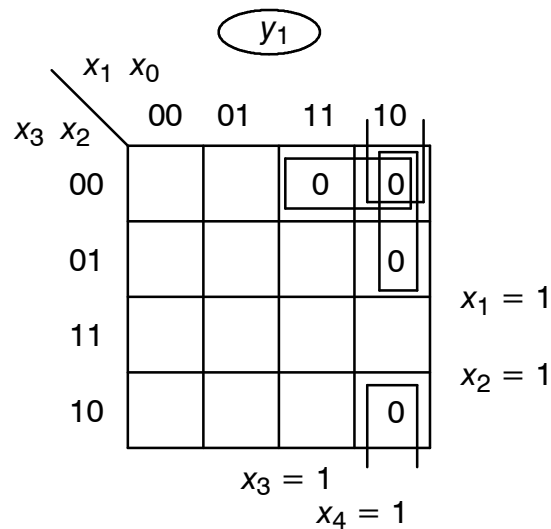
(1.4) Grouping four binary digits give one hex digit:

$x = 015C_{16}$
 $-x = FEA4_{16}$

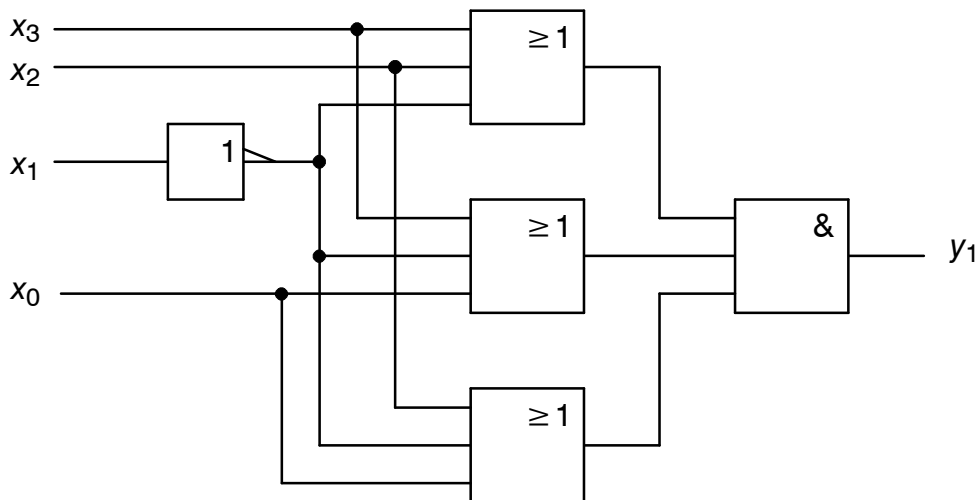
(1) = 8	(2) = 8	(3) = 8	(4) = 8	$\Sigma = 32$					
1.0 ≥ 27	1.3 ≥ 25	1.7 ≥ 23	2.0 ≥ 21	2.3 ≥ 19	2.7 ≥ 18	3.0 ≥ 17	3.3 ≥ 15	3.7 ≥ 13	4.0 ≥ 12

(2) Combinational Logic (Minimization)(2.1) Function $y_0 = F_0(x_0, x_1, x_2, x_3)$ 

$$y_0 = x_3 \cdot x_0'$$

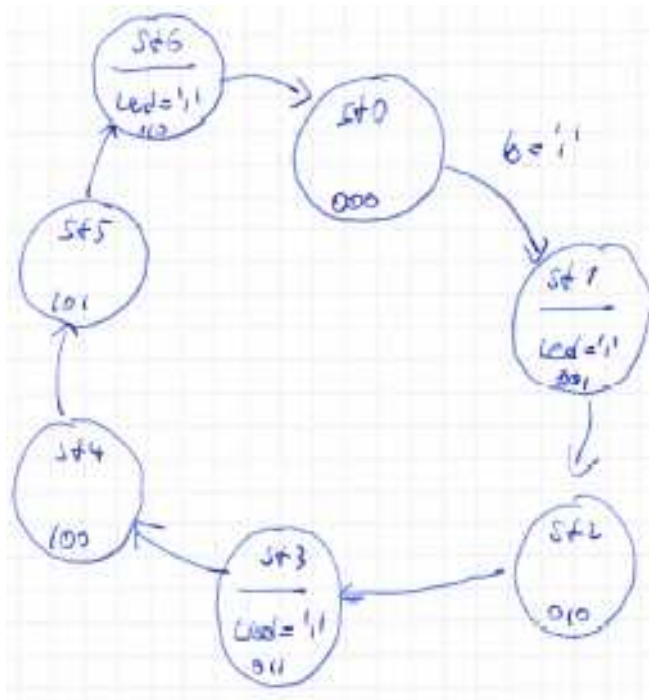
(2.2) Function $y_1 = F_1(x_0, x_1, x_2, x_3)$ 

$$y_1 = (x_3 + x_2 + x_1') \cdot (x_3 + x_1' + x_0) \cdot (x_2 + x_1' + x_0)$$



(3) Hardware State Machine (Moore Machine)

(3.1) State diagram



(3.2) 6 states \Leftrightarrow 3 FFs. However, if your state diagram is different other number of states are possible.

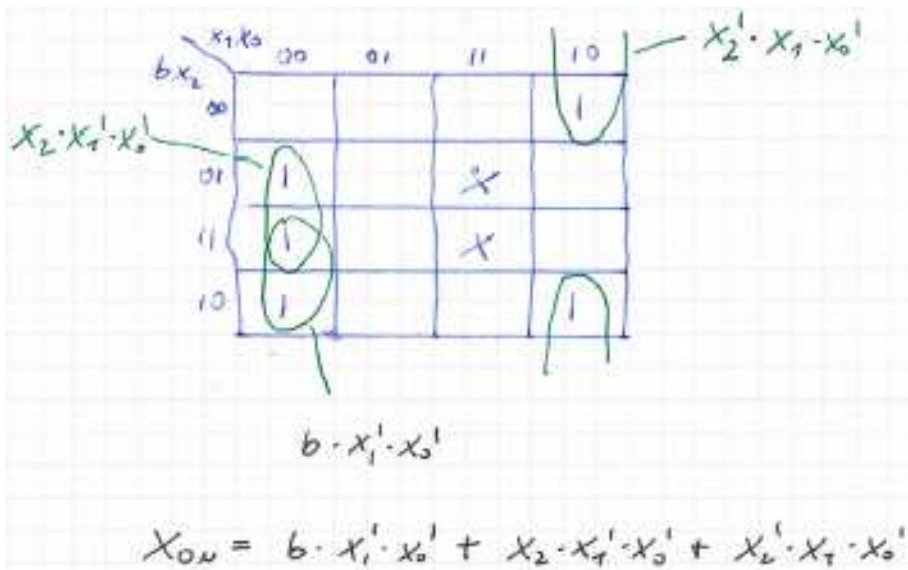
(3.3) Input logic, truth table

b	x_2	x_1	x_0	x_{2n}	x_{1n}	x_{0n}
0	0	0	0	0	0	0
0	0	0	1	0	1	0
0	0	1	0	0	1	1
0	0	1	1	1	0	0
0	1	0	0	1	0	1
0	1	0	1	1	1	0
0	1	1	0	0	0	0
0	1	1	1	X	X	X
1	0	0	0	0	0	1
1	0	0	1	0	1	0
1	0	1	0	0	1	1
1	0	1	1	1	0	0
1	1	0	0	1	0	1
1	1	0	1	1	1	0
1	1	1	0	0	0	0
1	1	1	1	X	X	X

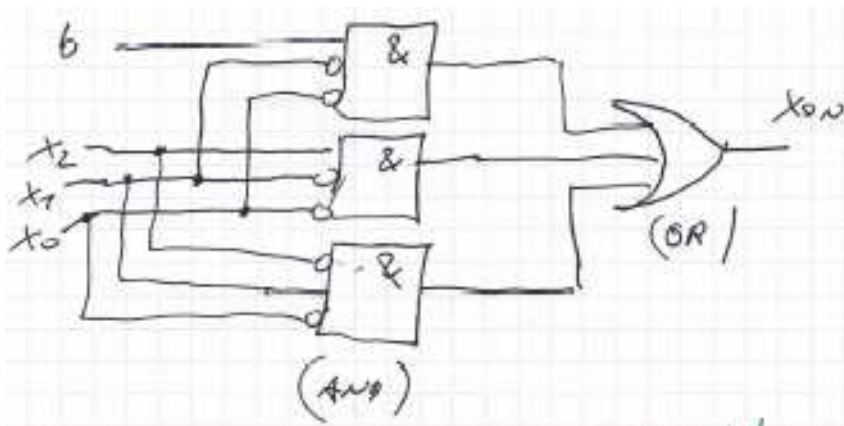
(never happens)

(never happens)

The truth table for the next state x_{0n} corresponds to the Karnaugh map:



The circuit for x_{0n} is as follows:



(3.4) Output logic, truth table:

x_2	x_1	x_0	led
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	X

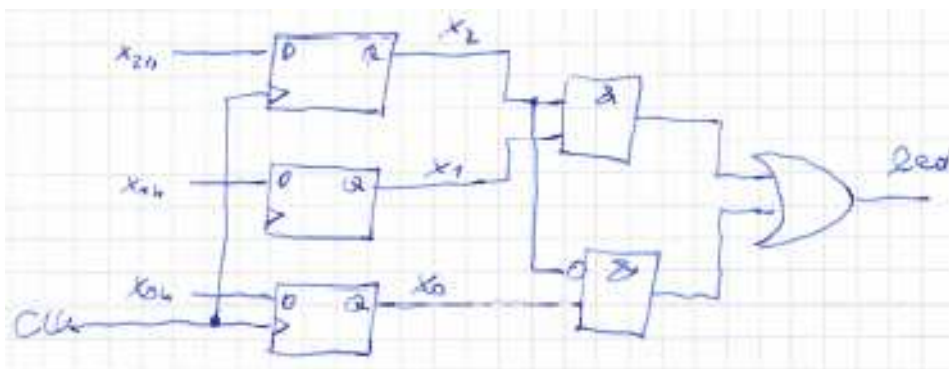
$x_2 \cdot x_0$

	$x_1 x_0$	00	01	11	10
x_2	0		1	1	
	1			1	

$x_2 \cdot x_1$

$led = x_2 \cdot x_1 + x_2 \cdot x_0$

matching the circuit schematics (only FFs and output logic):



(4) PicoBlaze Program “Moving Lights”

Attach program listing (with comments!) to your written exam.